Exam Advanced Logic

June 20th, 2017

Instructions:

- Put your student number on the first page and subsequent pages (not your name, in order to allow for anonymous grading).
- Do not use pencil (or an erasable pen) or a red pen to make your exam.
- Motivate all your answers.
- Please leave open 10 lines on the first page to give us room for writing down the points.
- Your exam grade is computed as min(10, (the sum of all your points + 10) divided by 10). For example, someone who would only fill in a student number would get a 1, while someone who would make questions 1-9 perfectly but would skip the bonus question would get min(10, (90+10)/10) = 10.
- If you want to inspect your exam after it has been graded, you can do so by making an appointment with Rineke Verbrugge.

Good luck!

1. **Induction (10 pt)** Let $\mathscr{L}_{\neg,\rightarrow}$ be a restricted language of propositional logic based on only operators $\neg$ and $\rightarrow$ (so without $\wedge$, $\vee$ and $\leftrightarrow$).
   (a) Give an inductive definition of the well-formed formulas of $\mathscr{L}_{\neg,\rightarrow}$.
   (b) Show by induction that $\{\neg, \rightarrow\}$ is functionally complete, i.e. that every well-formed formula P of propositional logic is equivalent to a formula P' in $\mathscr{L}_{\neg,\rightarrow}$. Prove this in three steps:
      i. Define P' by induction on well-formed formulas of propositional logic.
      ii. Show by induction that for every well-formed propositional formula P, the formula P' is in $\mathscr{L}_{\neg,\rightarrow}$.
      iii. Prove by induction that for every formula P of propositional logic, P' is logically equivalent to P.

2. **Three-valued logics (10 pt)** Using a truth table, determine whether the following inference holds in $\mathbf{Ł}_3$:

$$\models_{Ł_3} ((p \supset q) \supset p) \supset p$$

   Write out the full truth table and do not forget to draw a conclusion.

3. **Tableaus for FDE and related many-valued logics (10 pt)** By constructing a suitable tableau, determine whether the following inference is valid in **LP**. If the inference is invalid, provide a counter-model.

$$(\neg p \vee r) \wedge (\neg q \vee r) \vdash_{LP} \neg(p \vee q) \vee r$$

   NB: Do not forget to draw a conclusion from the tableau.

4. **Fuzzy logic (10 pt)** Determine whether the following holds in the fuzzy logic with $D = \{x : x \geq 0.9\}$. If so, explain why. If not, provide a counter-model and explain why not.

$$(p \rightarrow q) \rightarrow q \models_{0.9} p \vee q$$

5. **Basic modal tableau (10 pt)** By constructing a suitable tableau, determine whether the following is valid in K. If the inference is invalid, provide a counter-model.

$$\Diamond(\Diamond p \wedge \Diamond q) \vdash_K \Diamond\Diamond p \wedge \Diamond\Diamond q$$

   NB: Do not forget to draw a conclusion from the tableau.

6. **Normal modal tableau (10 pt)** By constructing a suitable tableau, determine whether the following tense-logical inference is valid in K^t_τ. If the inference is invalid, provide a counter-model.

$$\langle F\rangle[P]q \vdash_{K^t_\tau} [F][P]q$$

NB: Do not forget to draw a conclusion from the tableau.

7. **Completeness (10pt)** Let b be a complete open branch of a K_η-tableau, and let $I = \langle W, R, v\rangle$ be the interpretation that is *induced* by b. Show that the accessibility relation R of I is extendable, that is, for all $x \in W$, there is a $z \in W$ such that xRz.

8. **First-order modal tableau, variable domain (10 pt)** By constructing a suitable tableau, determine whether the following is valid in VK. If the inference is invalid, provide a counter-model.

$$\exists x\Diamond(Px \wedge Qx) \vdash_{VK} \Diamond\exists xPx \wedge \Diamond\exists xQx$$

NB: Do not forget to draw a conclusion from the tableau.

9. **Default logic (10 pt)** The following translation key is given:

$H(x)$	x has a homework average of 9.5 or higher
$F(x)$	x is a woman
$C(x)$	x is from China
$A(x)$	x studies AI
d	Yanjing

Consider the following set of default rules:

$$D = \left\{\delta_1 = \frac{H(x) : F(x)}{F(x)}, \qquad \delta_2 = \frac{C(x) : A(x)}{H(x)}, \qquad \delta_3 = \frac{A(x) : C(x)}{\neg F(x)}\right\},$$

and initial set of facts:

$$W = \{C(d), \neg C(d) \vee A(d)\}.$$

This exercise is about the default theory $T = (W, D)$; so you only need to apply the defaults to the relevant constant d.

(a) Of each of the following sequences, state whether it is a *process*; and if so, whether or not the process is *closed*, and whether or not it is *successful*. Briefly explain your answers.

   i. the empty sequence ( )
   ii. (δ_2)
   iii. (δ_2, δ_3)
   iv. $(\delta_2, \delta_3, \delta_1)$

(b) Draw the process tree of the default theory (W, D).

(c) What are the extensions of (W, D)?

Bonus; 10 pt This exercise compares the provable formulas of classical propositional logic and RM_3. Consider the language of RM_3 which contains the connectives $\neg, \vee, \wedge$ and $\supset$. Does the following statement hold for all wffs A in that language, where the first $\vdash$ stands for tableau provability in classical propositional logic?

"If $\vdash A$, then $\vdash_{RM_3} A$"

If yes, please explain exactly why the statement holds for all wffs A in the language of RM_3 If no, please provide a wff A and show in detail why that wff forms a counterexample to the statement.

As a reminder, you can find the RM_3 tableau rules for $\supset$ on the next page.

RM_3 tableau rules for $\supset$